

limb projected upon the Sun's disk; and, until the recent eclipse, this difficulty seemed more abstruse than it is now.

The fact that the intruding ray was only seen on this occasion in one valley of the Moon, and that the deepest of those visible upon the limb, would appear to corroborate the hypothesis that all the Moon's atmosphere of appreciable density is to be found secluded in her deepest valleys.

There is, however, another circumstance not to be overlooked in this interpretation, and which, combined with the last consideration, leaves, I think, nothing further to be explained; viz., that the extent of lunar atmosphere traversed by the incident ray at the cusp subtends a greater visual angle than at any other point on the limb; so that, when a deep lunar valley does appear at that point, the effect of refraction is exhibited under the most favourable circumstances.

It will not surprise me if it can be shown that my theory is untenable; but as far as the observations go, I respectfully offer them for consideration as good observations, and such as may possibly throw some light on the question whether the Moon has any atmosphere or not.

38, Harley Street, Cavendish Square.  
1874. November 10.

On the Inner Satellites of Uranus. By Prof. Edward S. Holden, U.S.N.

It is well known that Sir William Herschel suspected that the planet which he had discovered was accompanied by six Satellites.

These he numbered in their order, proceeding outwards from the planet:—

I	Period	5	days	21	hours
II	"	8	"	18	"
III	"	10	"	23	"
IV	"	13	"	11	"
V	"	38	"	01	"
VI	"	107	"	16	"

Of the existence of satellites II and IV there is no manner of doubt, since they were steadily observed by the elder Herschel from 1787, January 11, to 1810, May 25, and by his son in the years 1828 to 1832, as well as by Lamont, Struve, Lassell, and Newcomb since that time, and all of these observations have been consistent.

Satellites I, III, V, and VI have no such evidence in favour of their existence as the brighter ones II and IV.

From the *Philosophical Transactions*, 1815, where Herschel has collected all his observations of the satellites of *Uranus*, it appears that he supposes satellite I to have been observed on four different occasions, viz.:—

January 18, 1790

February 15, 1798

March 27, 1794

April 17, 1801

Satellite III was seen only twice, on March 26 and 27, 1794; while V and VI were suspected at various times.

Lassell, in 1847, discovered two satellites interior to Herschel's II, and we owe to him, and to his assistant Marth, a good series of observations of all four satellites. The periods, as determined by Lassell, are:

	d	h	m	s
Ariel =	2	12	29	20.7
Umbriel =	4	3	28	7.5
Titania =	8	16	56	25.6
Oberon =	13	11	6	55.4

It is evident that Herschel's II and IV are Lassell's *Titania* and *Oberon*.

Lassell's station at Malta was much better in regard to clearness of sky than Herschel's in England; his instrumental means were far superior, and the altitude of *Uranus* was greater at Malta in 1864 than in England in 1798, so that we must assume that if Lassell could not see Herschel's I, III, V, and VI, they do not exist.

In his report of his observations (*Memoirs, R.A.S.*, Vol. xxxvi), Lassell says that he repeatedly scrutinised the vicinity of the planet for the purpose of detecting faint satellites exterior to *Oberon*, and that he never suspected the existence of any such. Therefore, in the examination of Herschel's observations, I shall reject all those referring to satellites V and VI, and for the same reason I shall reject all those referring to satellite III; in this latter case we have the added testimony of five months' observations with the Alvan Clark refractor of the United States Naval Observatory at Washington.

There remains then of Herschel's observations only those of suspected interior satellites, which it will be profitable to examine.

Before selecting any of these observations for discussion, it is necessary to premise a few words in regard to Herschel's method of observation. On very few occasions he was able to faintly illuminate the wires of his micrometer for a determination of the position of *Oberon* and *Titania*, but *all* of his estimations of the position of any small objects interior to these had to be made in a perfectly dark field. Hence these estimations are liable to a large error of from 5 to 15 degrees in position-angle. Owing to the glare of the planet in the field of the telescope, Herschel

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found that he could seldom see *Oberon* nearer to the planet than  $23''\cdot6$ , while *Titania* was usually invisible at distances less than  $18''\cdot1$ .

Of course under ordinary circumstances *Ariel* and *Umbriel* could not be visible at all; but there were occasions when the fine polish of his mirror, or the good state of the atmosphere, permitted him to view objects even as close as  $10''$ . It was evidently impossible for him to see an interior satellite on two consecutive nights, and of this he was fully aware.

It was his habit to make his observations of the "first" and "second" satellites (*i.e.* of *Titania* and *Oberon*), and to map down all small stars near to the planet. On the next subsequent observing night he examined the spot where the planet had been, and was thus able to identify all small stars as such. In his printed observations the sketches of star-configurations are not given, but his remarks in full are quoted, followed by an "identification," as he calls it, of all suspected satellites. The patience and skill with which these identifications are carried out year after year are truly admirable, and they give a real value to that which without them would be simply a ponderous mass of useless material.

I have selected from his observations as printed, *all* cases where he has seen an object interior to *Titania*, sufficiently well to allow him to give an estimate of its position, excluding of course all cases where he has subsequently proved that such object was certainly a star.

The cases for examination are:—

1.	1787 February 10	Position of interior satellite	$135^\circ$
2.	1790 January 18	"following"	
3.	January 20		$315^\circ$
4.	1793 February 5		$250^\circ 57'$
5.	March 9		$205^\circ$
6.	1794 February 28		$66^\circ$
7.	March 27		
8.	1798 February 15		$5^\circ 11'$
9.	1801 April 17		$189^\circ$ distance $18''$

The elements which I have used are provisional ones, derived by Professor Newcomb from Lassell's Malta observations. They are amply adequate to the present enquiry.

From these elements I have computed the angle of position and distance of *Umbriel* and *Ariel* in each of the cases above set down, and compared these with Herschel's observations, as follows:—

1. "1787, February 10,  $8^h 57^m$ . The first satellite is about  $53^\circ$  n.p.  $8^h 33^m$  . . . . A supposed third is about  $45^\circ$  s.f. In a little more than four hours I saw the satellites go on with the planet, and also in their orbits. . . . No subsequent observation of the third was made."

On this date *Umbriel* was in  $P = 82^\circ$ , distant  $16''$ , and *Ariel* was n.p. Hence Herschel's "supposed third" was neither of the inner satellites.

2. "1790, January 18,  $9^h 32^m$ . There is a supposed third satellite about two diameters of the planet following, extremely faint and only seen by glimpses;  $1^h 6^m$  after I could not perceive it: a fourth is about  $70^\circ$  n.p."

"Two diameters" of the planet was about  $8''$ , and as Herschel usually counted his distances from the *limb* of the planet in his estimations, the distance from the centre would be about  $10''$ . *Umbriel* was in  $P = 124^\circ 23'$ , and distant  $13''\cdot9$ . *Ariel* was in  $P = 317^\circ 10'$ , distant  $10''\cdot96$ . " $1^h 6^m$  after" *Umbriel* was in  $P = 118^\circ 48'$ , and distant  $13''\cdot54$ ; i.e. nearer to *Uranus* by  $0''\cdot4$ . So far as the evidence goes, we may reasonably infer that Herschel had a glimpse of *Umbriel*. In the *Phil. Trans.* 1798, p. 271, Herschel, in referring to this observation, speaks of it as very certain, and supposes that the satellite might have been "11 or 12 degrees" from the parallel. The above identification, it is true, brings it 34 degrees from the parallel, but we must remember that, in the first place, the angle of position was merely an estimate, and secondly that "following" is here shown by Herschel himself to have been but a rough term, not indicating an angle of position of exactly  $90^\circ$ .

The "fourth satellite" of this night was proved to have been a star.

3. "1790, January 20,  $12^h 5^m$  . . . . a third satellite is  $45^\circ$  n.p. in a line with the planet and second satellite."

*Umbriel* had a position-angle of  $301^\circ$ , and was distant  $13''\cdot72$ : *Ariel* was n.f.; *Oberon* (the "second satellite") was in  $P = 324^\circ 08'$ , and we must again conclude that *Umbriel* was seen.

4. "1793, February 5,  $9^h 18^m$  . . . . a very small star is  $19^\circ 3'$  s.p. . . . There is no subsequent observations of the small star."

*Umbriel* was in  $P = 187^\circ 19'$  and *Ariel* was in n.p. Herschel's "small star" had a position-angle of  $250^\circ 57'$ , and hence it was neither *Ariel* nor *Umbriel*.

5. "1793, March 9,  $10^h 35^m$  . . . a third [satellite] is about  $65^\circ$  s.p."

*Umbriel* was n.p. and *Ariel* was n.f., and hence this observation refers to neither of them.

6. "1794, February 28,  $9^h 43^m$  . . . There is a small star . . . about  $24^\circ$  n.f."

Both *Ariel* and *Umbriel* were n.p. and hence the small object was a star.

7. "1794, March 27. A supposed third of this evening is preceding the first satellite, but nearer the planet. . . . The first satellite was  $79^{\circ}$  n.f."

*Titania* (the "first satellite") was in  $P=10^{\circ} 20'$ , distant  $34'' 43$ , while *Ariel* was in  $P=20^{\circ} 48'$ , and distant  $13'' 48$ , *Umbriel* being at this time s.f. Hence we must conclude that Herschel saw *Ariel*.

8. "1798, February 15,  $12^h 13^m$  . . . . position of the supposed fifth satellite" [which was really *Oberon*] " $84^{\circ} 49'$  n.f." . . . . at "about half the distance of the second satellite," [and between it and the planet Herschel saw what] "must have been an interior satellite, at its greatest northern elongation."

*Oberon* was in  $P=14^{\circ} 46'$ , distant  $32'' 00$ ; *Ariel* was in  $P=213^{\circ} 7'$ , distant  $5'' 06$ , and therefore invisible; *Umbriel* was in  $P=194^{\circ} 26'$ , distant  $18'' 99$ , and therefore in its most favourable position. Herschel says the interior satellite was between *Oberon* and the planet, and if this is so he did not see *Umbriel*. His account of this night's work (*op. cit.* pp. 332-3 and 359) is confused, and leads to the suspicion (no more) that an examination of the originals might prove his position of the interior satellite,  $180^{\circ}$  wrong, in which case *Umbriel* would have been seen. As it is, we must suppose the contrary.

9. "1801, April 17,  $10^h 30^m$ . There is a third satellite at a great angle south preceding: in the configuration it is marked exactly in apposition to the second, and at half the distance of the first. . . . The third by the configuration was  $81^{\circ}$  s.p." On the next night Herschel examined the place where the planet was on April 17, and found no star in the former place of the third satellite.

Herschel's satellite was in  $P=189^{\circ}$ , distant  $18''$ . *Umbriel* was in  $P=191^{\circ} 27'$ , distant  $21'' 18$ . Hence Herschel saw *Umbriel*.

The above are all the cases which a careful examination of the printed observations suggests for discussion; and it is to be remarked that of the four cases where Herschel supposed he saw the interior satellites, three have been verified fully, and a reasonable suspicion exists that the fourth may have been likewise a veritable observation of *Umbriel*. A reference to the originals would probably settle most of the doubts which have arisen.

We may conclude then that the elder Herschel was in truth the discoverer of *Ariel* and *Umbriel*, as well as of *Titania* and *Oberon*, but that he was unfortunately prevented from identifying



the inner satellites because his telescope could not show them on two successive nights.

It is to be noted that Sir John Herschel never caught a glimpse of them during his examination of *Uranus* with the same telescope in 1828-1832; the extreme difficulty of these objects makes us wonder at the marvellous skill and patience manifested by the elder Herschel in this difficult research.

It would be an interesting and useful research to endeavour to explain Herschel's observations, of the III, V, and VI satellites, and to show that these were observations of small stars. This research I hope to execute upon the return of Professor Watson, and Dr. C. H. F. Peters, from their respective journeys for the purpose of observing the Transit of *Venus*. Both of these astronomers have the most extended and minute maps of all the small stars in the region where *Uranus* was in 1787-1801; and a careful examination of tracings of these maps and of Herschel's observations could not fail to throw some light upon the supposed discovery of satellites III, V, and VI.

The next observations of these subjects were by Lassell and O. Struve in 1847.

Lassell's observations are given in the *Monthly Notices* of the Royal Astronomical Society, vol. viii. p. 44. I have compared these with the theory as below:—

Date.	Lassell's Position of Satellite.	Position of <i>Umbriel</i> .	Position of <i>Ariel</i> .
1. 1845 Oct. 5	324°1	bet. 0° and 90°	bet. 0° and 180°
2. 1847 Sep. 14	350	bet. 0° and 90°	P = 345°·4    Δ = 13''·96
3. 1847 Sep. 27	326		P = 320°·1    Δ = 10''·45
4. 1847 Sep. 29	336	P = 173° 3	P = 1°·3    Δ = 13''·27
5. 1847 Oct. 1	348    Δ = 18''·44	P = 345° 8',    Δ = 19''·10	
6. 1847 Nov. 6	80    Δ = 10	{ P = 141° 36', Δ = 15''·4	P = 350°·1    Δ = 13''·89
"    "    }	349    Δ = 11		

Hence it is plain that Lassell saw *Ariel* on September 14, September 27, and November 6, and possibly on September 29. *Umbriel* was seen on October 1.

Struve's observations are given in the *Monthly Notices*, R.A.S. vol. viii. p. 46.

Date.	P. of <i>Umbriel</i> (Struve).	P. of <i>Umbriel</i> .
1. 1847 Nov. 1	P = 194    Δ = 17·8	P = 186° 47'    Δ = 16''·59
2.    "    Nov. 28	203·6    17·0	bet. 0° and 90°
3.    "    Dec. 9	218·6    13·7	P = 152° 54'    Δ = 17''·4 but <i>Ariel</i> bet. 270° and 360°
4.    "    Dec. 10	180·1    17·0	P = 169° 38'    Δ = 19''·3
5. 1848 Jan. 25	202·0    18·0	bet. 0° and 90°

Hence Struve may have seen *Umbriel* November 1, and December 1847.

In 1871 observations of *Ariel* and *Umbriel* were made at Bothkamp with a telescope of 12-inches aperture, and compared with Marth's ephemeris for that year. The positions of *Ariel* differ from their predicted positions by a large angle, nearly  $180^\circ$ ; while the positions of *Umbriel* agree well.

*Ariel*, however, is much the brighter of the two inner satellites; and as it evidently was not seen at all, it becomes probable that a small star was mistaken for *Umbriel* on the five nights of observation.

This supposition is strengthened when we consider that the Bothkamp observers found *Titanian* and *Oberon* difficult objects, which they certainly are not to any telescope which will show *Ariel* or *Umbriel*. Lassell estimates that *Oberon* and *Titanian* are twice as bright intrinsically as either of the inner satellites, and this estimate is probably not too high.

We may then fairly claim that Sir William Herschel saw all four of the satellites of *Uranus*; that Lassell discovered independently and first of any of his contemporaries the two faint satellites; while Struve probably saw *Umbriel* on one or two occasions; and that the inner satellites have not all been seen with any telescopes, save the 20- and 40-foot reflectors of Herschel, the telescopes of Mr. Lassell (2- and 4-foot reflectors) and by the Clark refractors at Washington.

The great refracting telescope of Dr. Henry Draper and the Clark refractor of Chicago and Mr. Newall's Cooke refractor have never been used upon these objects so far as I know, although all three are undoubtedly adequate to the research.

1874. U.S. Naval Observatory, Washington.

In reference to the foregoing Paper, Mr. Lassell has written to the Secretary of the Society the following letter:—

Dear Sir,—At the Meeting of the Royal Astronomical Society on the 13th inst., when Professor Holden's Paper on the Inner Satellites of *Uranus* was partly read, I took exception to some of its statements, and have in consequence been favoured with the perusal of a copy of it as set up in type. I respectfully request your insertion in the same notice of the following remarks:—

First, I cannot for a moment admit the correctness of Professor Holden's statement, that—"We may conclude then that the elder Herschel was in truth the discoverer of *Ariel* and *Umbriel*, as well as of *Titanian* and *Oberon*." From a less distinguished authority than that of an astronomer writing from the Washington Observatory, the assertion might have passed unnoticed, from a conviction that it would not obtain credence;